$$\frac{E_y^2}{E_{y_0}^2} + \frac{E_z^2}{E_{z_0}^2} = 1$$

Thus the electromagnetic wave is said to be elliptically polarized wave.
If $E_{y_0} = 0$ or $E_{z_0} = 0$ then the electric field is said to be linearly polarized or plane polarized. If $E_{y_0} = E_{z_0}$ then the electric field is said to be circular polarized

## 5.3 Electron plasma waves in warm plasma :-

Consider a warm homogeneous collision less plasma describe by a scalar pressure and consisting of electrons and ions with no external applied field. Ions because of their heavier mass are assume to be immobile forming a uniform neutralizing background. Now the eqn. governing the electron fluid motion are

$$\frac{\partial n_e}{\partial t} + \vec{\nabla}\cdot(n_e \vec{u}_e) = 0 \quad \text{(Continuity eqn)} \quad (1)$$

$$m_e n_e \left(\frac{\partial}{\partial t} + \vec{u}_e\cdot\vec{\nabla}\right)\vec{u}_e = -e n_e \vec{E} - \vec{\nabla} p \quad \text{(momentum eqn)} \quad (2)$$

$$\epsilon_0 (\vec{\nabla}\cdot\vec{E}) = e(n_i - n_e) \quad \text{(Poisson eqn)} \quad (3)$$

The addition eqn we need is the eqn of the state

$$p = C\rho_m^{\gamma} \quad (4)$$

$$\frac{\vec{\nabla} p}{p} = \gamma \frac{\vec{\nabla}\rho_m}{\rho_m} = \gamma \frac{\vec{\nabla} n_e}{n_e} \quad (5)$$

Since $p = n_e k_B T_e$ ; $\vec{\nabla} p = \gamma k_B T_e \vec{\nabla} n_e$ (6)

To study the case of wave motion in one dimension we can take degrees of freedom $f = 1$ & $\gamma = 1 + 2/f = 3$. In one dim. all the gradients in above equations become derivatives w.r.t $x$ only. We assume small amplitude waves so that perturbations in plasma parameters are small as compared to their equilibrium values. Let

$E_1$ $u_1$ $n_1$

$$n_e = n_0 + n_1 \quad , \quad \vec{u}_e = \vec{u}_0 + \vec{u}_1 \quad , \quad \vec{E} = \vec{E}_0 + \vec{E}_1 \quad \text{--- (F)}$$

where the subscript 0 refers to equilibrium values and subscript 1 refers to the perturbed part. For a uniform neutral plasma at rest in the equilibrium state,

$$\vec{E}_0 = \vec{u}_0 = 0 , \quad \vec{\nabla} n_0 = 0 , \quad \frac{\partial n_0}{\partial t} = \frac{\partial \vec{u}_0}{\partial t} = \frac{\partial \vec{E}_0}{\partial t} = 0$$

now substituting the perturbation expansion (F) in eqn (1) to (3) and using (6) we get -

$$\frac{\partial n_1}{\partial t} + n_0 \vec{\nabla} \cdot \vec{u}_1 = 0 \quad \text{--- (8)}$$

$$m_e n_0 \frac{\partial \vec{u}_1}{\partial t} = -e n_0 \vec{E}_1 - 3 k_B T_e \vec{\nabla} n_1 \quad \text{--- (9)}$$

$$\epsilon_0 \vec{\nabla} \cdot \vec{E}_1 = -e n_1 \quad \text{--- (10)}$$

where we have assumed the changes $n_1, \vec{u}_1, \vec{E}_1$ due to perturbations to be small quantities of first order and neglected higher order smaller terms.

Considering one dimensional case & assuming that all the perturbation quantities vary as $\exp[i(kx - \omega t)]$, we get from eqn (8) to (10)

$$-i\omega n_1 + n_0 i k u_1 = 0 \quad \text{--- (11)}$$

$$-i\omega m_e n_0 u_1 = -e n_0 E_1 - 3 k_B T_e \, i k n_1 \quad \text{--- (12)}$$

$$\epsilon_0 i k E_1 = -e n_1 \quad \text{--- (13)}$$

$$\begin{bmatrix} 0 & n_0 i k & -i\omega \\ e n_0 & -i\omega m_e n_0 & 3 k_B T_e i k \\ \epsilon_0 i k & 0 & e \end{bmatrix} = 0$$

$\Rightarrow -n_0 ik\left(e^2 n_0 + 3\epsilon_0 k_B T_e k^2\right) + (-i\omega)\left[-\epsilon_0 k \omega m_e n_0\right] = 0$

$\Rightarrow \omega^2 \epsilon_0 m_e = e^2 n_0 + 3\epsilon_0 k_B T_e k^2$

$$\Rightarrow \omega^2 = \frac{e^2 n_0}{\epsilon_0 m_e} + \frac{3k_BT_e}{m_e}k^2 \quad \text{——— (14)}$$

$$\Rightarrow \omega^2 = \omega_{pe}^2 + \frac{3}{2}k^2 V_{Th}^2 \quad \text{——— (15)}$$

where $\omega_{pe} = \sqrt{\frac{e^2 n_0}{\epsilon_0 m_e}}$ & $V_{Th} = \sqrt{\frac{2k_BT_e}{m_e}}$ is the thermal velocity.

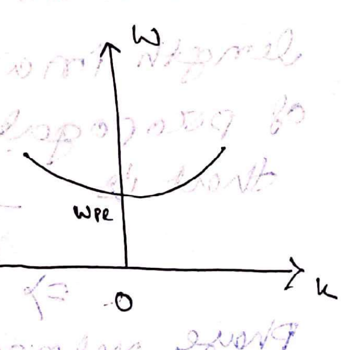
This dispersion relation is known as Bohm-Gross dispersion relation for longitudinal electron plasma wave. The wave frequency $\omega$ now depends on wave number $k$ & group velocity is not zero. That means that the wave is now a propagating wave.

The nature of the dispersion relation (15) is ~~known~~ shown graphically in the figure. Note that the electron plasma waves are basically constant frequency type but become of constant velocity type at large $k$. In terms of Debye length $\lambda_D \left(= \sqrt{\frac{\epsilon_0 k_B T_e}{n_0 e^2}}\right)$ the dispersion relation (15) may be written as

$$\omega_0 = \omega_e^2\left(1 + 3k^2\lambda_D^2\right) \quad \text{——— (16)}$$

It shows that the thermal perturbation to plasma becomes important when $k\lambda_D \sim 1$ meaning that wavelength of perturbation $\sim \lambda_D$.

The phase velocity of the wave is




$$v_p = \frac{\omega}{k} = \sqrt{\frac{3}{2}}\, V_{Th} \cdot \frac{\omega}{\sqrt{\omega^2 - \omega_{pe}^2}} \quad \text{——— (17)}$$

from (15) we have

$$2\omega \frac{d\omega}{dk} = 3k V_{th}^2$$

$$\Rightarrow v_g = \frac{d\omega}{dk} = \frac{3}{2}\cdot\frac{k}{\omega}V_{th}^2 = \frac{3}{2}V_{th}^2 \cdot \frac{1}{v_p}$$

$$= \sqrt{\frac{3}{2}}\, V_{th} \cdot \frac{\sqrt{\omega^2 - \omega_{pe}^2}}{\omega} \quad \text{--- (18)}$$

from the dispersion relation (15) it may be noted that at large $k$ (small $\lambda$) $\omega^2 >> \omega_{pe}^2$ then from (18) $v_g \approx \sqrt{\frac{3}{2}}\, V_{th}$.

This means that at large $k$ information travels essentially at thermal velocity. But at small $k$ (large $\lambda$) $v_g < V_{th}$ i.e the information travels more slowly than the thermal speed.

## (*) Group velocity :-

If a number of wave of different wave length are moving with different velocity in a medium. Then the observed velo. is the velocity of wave packet. This is called group velocity. we define the group velocity as $v_g = \frac{d\omega}{dk}$ which is not exceeds the velocity of light $c$.

## (*) phase velocity :-

When a single wave of definite wave length in a medium then its velocity of propogation are called phase velocity that is $\frac{d}{dt}(kx - \omega t) = 0$

$$\Rightarrow \frac{dx}{dt} = \frac{\omega}{k}$$ which is called Phase velocity. which often exceeds the velocity of light $c$.

If $\frac{\omega}{k}$ +ve the wave moves right +ve x direction as forward.
If $\frac{\omega}{k}$ -ve " " " left

## Plasma oscillation in one dimensional drifting plasma :-

Many plasma applications involve plasma drifting w.r.t grids or others stationary many reference objects. In space charge disturbances in such a moving media are excited and have several properties in contrast to ~~text~~ space charge disturbances in a plasma at rest and their study introduces some general aspect of plasma waves. In addition to all assumption made in the previous problem that is when we have studied the plasma oscillations.

We have assume that plasma is drifting with const velocity in the same direction, but no force in the x-direction

Therefore, $v_e = v_0 + v_1$

Plasma oscillation in this situation are also analysed with the continuity eqn for electron in one dimension as

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot (n_e v_e) = 0 \quad \text{--- (1)}$$

The momentum transfer eqn for electron in one dimension neglecting the collision term is given by

$$\frac{\partial v_{ex}}{\partial t} + \left(v_{ex} \frac{\partial}{\partial x}\right) v_{ex} = -\frac{e}{m_e} E_x \quad \text{--- (2)}$$

The poisson's eqn in one dimensions given by —

$$\frac{\partial E_x}{\partial x} = 4\pi e (n_0 - n_e) \quad \Big| \; \epsilon_0 = \frac{1}{4\pi} \quad \text{--- (3)}$$

again assume that the density, velocity electric field are perturbed slightly since $v_0 \neq 0$, The eqn (1) - (3) can be solved by neglecting the terms of 2nd order in the perturbation quantity and assuming a static harmonic time dependent perturbation as —

$$n_e = n_0 + n_1 e^{i(kx-\omega t)} \qquad n_1 \ll n_0 \quad \text{—} (4)$$

$$v_e = v_0 + v_1 e^{i(kx-\omega t)} \qquad v_1 \ll v_0 \quad \text{—} (5)$$

$$E = E_1 e^{i(kx-\omega t)} \hat{x} \quad \text{—} (6)$$

neglecting the product of perturbation quantities as being 2nd order from eqns (1)–(3) we get

$$-i\omega n_1 + n_0 ik v_1 + v_0 ik n_1 = 0$$

$$-i\omega v_1 + v_0 ik v_1 = -\frac{e}{m_e} E_1 \quad \text{—} (7)$$

$$ikE_1 = -4\pi e n_1 \quad \text{—} (8)$$

$$\text{—} (9)$$

The above three eqns are homogeneous linear algebraic eqn. The nontrivial solutions of these eqns is determined by setting the determinant of the co-efficients of $n_1, v_1, E_1$ is equal to zero. Then we have —

$$\begin{vmatrix} -i\omega + v_0 ik & n_0 ik & 0 \\ 0 & -i\omega + v_0 ik & \frac{e}{m_e} \\ 4\pi e & 0 & ik \end{vmatrix} = 0$$

$$= (-i\omega + v_0 ik)(-i^2 k\omega + v_0 i^2 k^2) - n_0 ik\left(-\frac{e^2 4\pi}{m_e}\right) = 0$$

$$\Rightarrow i^3 k\omega^2 - i^3 \omega v_0 k^2 - i^3 k^2 v_0 \omega + i^3 k^3 v_0^2$$

$$+ \frac{4 n_0 e^2 \pi i k}{m_e} = 0$$

$$\omega_{pe} = \sqrt{\frac{n_0 e^2}{\epsilon_0 m_e}}$$

$$\Rightarrow -\cancel{k}\omega^2 + \omega v_0 k + k v_0 \omega - \cancel{k} k^2 v_0^2$$

$$+ \frac{4 n_0 e^2 \pi}{m_e} = 0$$

$$\Rightarrow -(\omega - k v_0)^2 + \frac{4 n_0 e^2 \pi}{m_e}$$

$$\Rightarrow (\omega - k v_0)^2 = \frac{4\pi e^2 n_0}{m_e} = \omega_{pe}^2$$

This is the dispersion relation for plasma of space charge wave associated with drifting plasma or an electron beam.

A co-ordinate transformation of the form $x \to x' - v_0 t'$

$t \to t'$

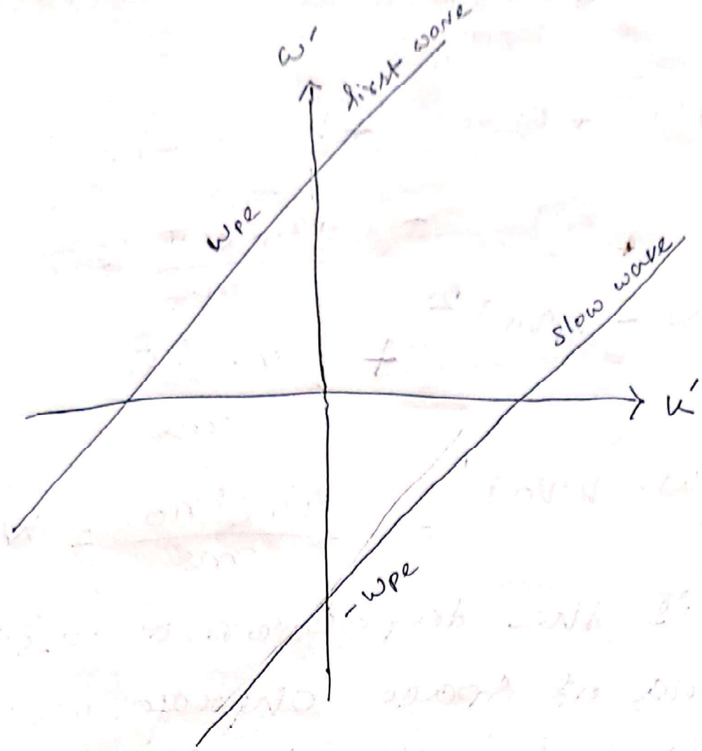
shows that the frequency and the wave number transformed to

$\omega \to \omega' + v_0 k'$ , $k \to k'$

and the above dispersion relations becomes

$$\omega'^2 = \omega_{pe}^2 \qquad \text{———} \quad (10)$$

Thus in the frame when electrons are at rest the disturbance is at the plasma frequency recovering the plasma oscillations as a stationary oscillations. Plasma oscillation in a one dimentional drifting plasma in the natural oscillation in plasma transported at a drifting velocity. the dispersion relation (10) is shown in the following figure —




Wave energy :—

The concept of wave energy simply means that if the system is in a state of energy $W_0$ Then the energy of the system $W = W_0 + W_1$ is smaller than $W$ when a wave of energy $W_1$ is launched then the total energy of the system can never be negative. The wave energy is composed of it electromagnetic energy

$\frac{E^2 + B^2}{8\pi}$ and also the polarization energy $W_{pe}$. If $W_{pe}$ is negative, The wave energy can also be negative in some cases. To observe how $W_{pe}$ be negative, we consider a small perturbation in the velocity of localised ~~stor~~ clamp of electrons as

$\vec{v}_1 = v_1 \hat{x}$